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1994 J. Phys. A: Math. Gen. 27 4189

(http://iopscience.iop.org/0305-4470/27/12/024)

J. Phys. A: Math. Gen. 27 (1994) 4189–4202. Printed in the UK

Radiation emission by a set of ultrarelativistic charged particles in a scattering medium

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Received 1 June 1993

Abstract. The bremsstrahlung of a system of classically fast charged particles which do not interact with each other but which do undergo multiple elastic scattering by randomly positioned atoms of a medium is studied. We derived the spectrum of the bremsstrahlung of such particles through a systematic kinetic analysis of the radiation process in the medium. It is shown that the spectral distribution of the emission energy of the bremsstrahlung depends significantly on both the characteristics of the scattering of the particles in the medium and the parameters characterizing the initial set of the particles.

1. Introduction

The bremsstrahlung of classically fast charged particles in a scattering medium was first studied by Landau and Pomeranchuk [1, 2]. They derived an expression for the spectral energy density of the bremsstrahlung. They pointed out that the bremsstrahlung intensity was suppressed at low frequencies by multiple elastic collisions of the irradiating particle with the atoms of the medium (the Landau–Pomeranchuk effect). Migdal [3] derived a quantitative theory for the bremsstrahlung of such a particle by averaging the spectrum of the radiation energy over all possible particle trajectories in an amorphous medium. The method proposed by Migdal for calculating the spectrum of the bremsstrahlung of a classically fast particle in a medium was developed further [4–7] during research of the dispersion properties of the scattering medium [4, 6], its boundaries [5], and inelastic processes which occur in the medium [6, 7].

However, only the radiation of an individual particle was studied in [1–7]. In many cases ([8, 9] for example), the source of the bremsstrahlung by fast particles moving through a scattering medium is a set of radiating particles. In addition, there is general physics interest in a study of the bremsstrahlung of a system of charged particles in a medium, since in this case an interference mechanism as well as the collisional mechanism forms the radiation spectrum. As a result, the frequency distribution of the bremsstrahlung and the dependence of the bremsstrahlung intensity on the thickness of the medium and on parameters characterizing the scattering of the particles in the medium are markedly different from those in the case of an individual irradiating particle.

In the present paper we research the bremsstrahlung of a system of classically fast charged particles which do not interact with each other but which do undergo multiple

0305-4470/94/124189+14$19.50

elastic scattering by randomly positioned atoms of the medium. We derived the spectrum of the bremsstrahlung of such particles through a systematic kinetic analysis of the radiation process in the medium. It is shown that the spectral distribution of the emission energy of the bremsstrahlung depends significantly on both the characteristics of the scattering of the particles in the medium and the parameters characterizing the initial set of the particles.

If a set of identical particles is present, the spectrum differs from that in the case of the individual radiating particle [1–3] in being very non-monotonic and in having at least one extremum, which result from interference of the wave emitted by the individual particles. In the limit of very low frequencies, the bremsstrahlung of the set of identical particles in a medium is formed under conditions corresponding to complete coherence of the individual radiators, while in the extreme short-wave part of the spectrum the bremsstrahlung intensity is proportional to the number of particles. We analyse in detail the radiation by a pulsed beam of identical charged particles and also the bremsstrahlung of a high anisotropic point source of ultrarelativistic radiators. It is shown that in this case the bremsstrahlung spectrum has a maximum, and that this maximum is unique. The value of the radiation energy at this maximum and also the shape of this maximum depend strongly on the characteristics of the scattering medium and also on parameters which specify the initial beam of the particles.

We analyse the radiation emission by the set of non-identical particles. It is shown that the differences in electrodynamic characteristics of irradiating particles such as mass, value of charge, and energy lead to suppression of the interference mechanism forming the bremsstrahlung spectrum. The spectral distribution of the bremsstrahlung by an ultrarelativistic electron–positron pair in a scattering medium is analysed in detail. We show that in this case the differences of the charges of the irradiating particles leads to a decrease in the value of the emission energy in the long-wave region of the radiation spectrum. Moreover, under some conditions there is a point of overbending in the spectral distribution of the bremsstrahlung of an electron–positron pair in a scattering medium.

2. Statement of the problem. Two-time distribution function in the *k*-representation

We consider the system of charged ultrarelativistic ($E_\mu \gg m_\mu$) classically fast ($E_\mu \gg \omega$ is a radiation frequency) particles which do not interact with each other (E_μ, m_μ and e_μ are the energy, the mass and the charge of the particle μ; $\hbar = C = 1$). These particles enter a homogeneous, semi-infinite, amorphous scattering medium. In the initial period $t=0$, particles are located at the points $\boldsymbol{r}_{01}, \boldsymbol{r}_{02}, \ldots, \boldsymbol{r}_{0N}$ and have the velocities $\boldsymbol{v}_{01}, \boldsymbol{v}_{02}, \ldots, \boldsymbol{v}_{0N}$, and $v_0 = [1-(m_\mu/E_\mu)^2]^{1/2}$, and they are directed under the angle $|\Delta_\mu| \ll 1$; $\mu = 1, \ldots, N$ to the $\boldsymbol{e}_z$ vector (vector of the inward normal to the boundary of the medium). Let the characteristic longitudinal size of the beam $l_B = \max_{\mu,\nu}\{|(\boldsymbol{r}_{0\mu} - \boldsymbol{r}_{0\nu})_z|\}$ be such that $l_B v_0^{-1}$ is small compared to the time T when the particles move in the medium.

The spectral distribution of the energy radiated by these particles is

$$\frac{\mathrm{d}\varepsilon_\omega}{\mathrm{d}\omega} = \frac{e^2\omega^2}{4\pi^2} \sum_{\mu,\nu=1}^{N} \int \mathrm{d}\Omega_n \int \mathrm{d}t_1 \int \mathrm{d}t_2 \exp(-\mathrm{i}\omega t_1 + \mathrm{i}\omega t_2) \times [\boldsymbol{n} \times \boldsymbol{j}^\mu_{i\to f}(\boldsymbol{k}, t_1)][\boldsymbol{n} \times (\boldsymbol{j}^\nu_{i\to f}(\boldsymbol{k}, t_2))^*] \tag{1}$$

where N is the number of particles, $\boldsymbol{k}$ is the wave vector of the radiation field, $\mathrm{d}\Omega_n$ is an element of the solid angle in the direction $\boldsymbol{n}=\boldsymbol{k}/k$, $k=\omega$, and $j^{\mu}_{i\to f}(\boldsymbol{k}, t)$ is a matrix of the current of the transition between the states i and f in the momentum representation. The integration in (1) is over the time spent by the particle in the medium.

If we ignore the interaction between particles, the function $j^{\mu}_{i\to f}(\boldsymbol{k}, t)$ is proportional to the Fourier component of the one-particle density matrix $\rho^{\mu}(\boldsymbol{r}_{\mu}, \boldsymbol{r}'_{\mu}; \boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_4)$, which depends on the coordinates of particle μ and also on the radius vectors $\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_4$. The latter specify the positions of the scattering centres in the medium (4 is the number of the scatterers).

To calculate the observed spectral distribution of the radiation energy of the particles in the medium, $\mathrm{d}E_{\omega}/\mathrm{d}\omega$, we must average (1) over all possible trajectories of the particles in the scattering medium [3]. To do this we need to find the expectation value (over all $\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_4$) of a bilinear combination of the density matrices $\rho^{\mu}(\boldsymbol{r}_{\mu}, \boldsymbol{r}'_{\mu}; \boldsymbol{R}_1, \ldots, \boldsymbol{R}_4)$ and $\rho^{\nu}(\boldsymbol{r}_{\nu}, \boldsymbol{r}'_{\nu}; \boldsymbol{R}_1, \ldots, \boldsymbol{R}_4)$. Multiplying the equations of motion for the operators $\rho^{\mu}(\boldsymbol{r}_{\mu}, \boldsymbol{r}'_{\mu}; \boldsymbol{R}_1, \ldots, \boldsymbol{R}_4)$ and $\rho^{\nu}(\boldsymbol{r}_{\nu}, \boldsymbol{r}'_{\nu}; \boldsymbol{R}_1, \ldots, \boldsymbol{R}_4)$ from the right and left by the matrices $\rho^{\nu}(\boldsymbol{r}_{\nu}, \boldsymbol{r}'_{\nu}; \boldsymbol{R}_1, \ldots, \boldsymbol{R}_4)$ and $\rho^{\mu}(\boldsymbol{r}_{\mu}, \boldsymbol{r}'_{\mu}; \boldsymbol{R}_1, \ldots, \boldsymbol{R}_4)$, respectively, and then summing the result, we find the following equations for the operator $\hat{\mathscr{P}}$, which is the product of ρ^{μ} and ρ^{ν}:

$$\mathrm{i}\frac{\partial\hat{\mathscr{P}}}{\partial\tau}=[\hat{H}^{\mu}, \hat{\mathscr{P}}] \qquad \mathrm{i}\frac{\partial\hat{\mathscr{P}}}{\partial t}=[\hat{H}^{\mu\nu}, \hat{\mathscr{P}}]. \tag{2}$$

The Hamiltonian $\hat{H}^{\mu}$ acts on the variables $\boldsymbol{r}_{\mu}$, the Hamiltonian $\hat{H}^{\mu\nu}$ acts on the variables $\boldsymbol{r}_{\mu}$ and $\boldsymbol{r}_{\nu}$, and we have $\tau=t_1-t_2$ and $t=t_2$. In the problem of the radiation by a charged particle, the quantity τ is the timescale of the radiation formation (the coherence time), and t is the time at which the radiation is emitted [3].

We expand the operator $\hat{\mathscr{P}}$ in (2) in a complete set of plane waves $u^{\lambda}_{p}\exp(\mathrm{i}\boldsymbol{p}\boldsymbol{r})$ ($\boldsymbol{p}$ is the momentum of the particle, and λ is the spin variable [10]), and we take an average over the positions of the scatterers in the medium in the resulting equations. Ignoring the 'mixing' of the spin components of the wavefunctions caused by scattering centres (this simplification is legitimate for ultrarelativistic particles [11]), we then find the following expression for $F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)$, the coefficients in the expansion of the operator $\hat{\mathscr{P}}$ (we are omitting the spinor indices):

$$\begin{aligned}\mathrm{i}\frac{\partial}{\partial\tau}\langle F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle &- (E_{p_1}-E_{p_2})\langle F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle \\ &= \sum_{g}(\langle V^{\mu}(g)F(\boldsymbol{p}_1+\boldsymbol{g}, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle \\ &\quad - \langle V^{\mu}(g)F(\boldsymbol{p}_1, \boldsymbol{p}_2-\boldsymbol{g}, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle)\end{aligned} \tag{3}$$

$$\begin{aligned}\mathrm{i}\frac{\partial}{\partial t}\langle F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle &- (E_{p_1}-E_{p_2}+E_{p_3}-E_{p_4})\langle F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle \\ &= \sum_{g}(\langle V^{\mu}(g)F(\boldsymbol{p}_1+\boldsymbol{g}, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle \\ &\quad - \langle V^{\mu}(g)F(\boldsymbol{p}_1, \boldsymbol{p}_2-\boldsymbol{g}, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle \\ &\quad + \langle V^{\nu}(g)F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3+\boldsymbol{g}, \boldsymbol{p}_4; t, t+\tau)\rangle \\ &\quad - \langle V^{\nu}(g)F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4-\boldsymbol{g}; t, t+\tau)\rangle)\end{aligned} \tag{4}$$

where

$$V^{\mu}(g)=\sum_{\alpha=1}^{4} U^{\mu}(g)\exp(-igR_{\alpha})$$

$U^{\mu}(g)$ is a Fourier component of the interaction potential of the scattering centre which is situated at the point R_{α} with the particle μ, E_p is the energy of the particle with a momentum p, and the vectors p_1, p_2, p_3, p_4 are related to the momenta of the particles μ and ν by means of the expressions

$$p_{1,2}=p_{\mu}\mp k/2 \qquad p_{3,4}=p_{\nu}\pm k/2.$$

The angular brackets denote the average over the positions of the scatterers.

Equations (3) and (4) constitute a system of integrodifferential equations which are not closed with respect to the unknown function, which depends on two time variables. The latter circumstance makes the calculation of the correlation function on the right-hand sides of (3) and (4), and also the derivation function, far more complicated than in the case of ordinary one-time problems of kinetic theory [12]. However, by virtue of the very formulation of the problem of the emission by a charged particle in a medium, the times t and τ satisfy the inequality $\tau \ll t$: the radiation must be emitted during the time the particle spends in the medium. To first order in the parameter $\tau t^{-1} \ll 1$ we can then ignore the dependence of the function $\langle F(p_1, p_2, p_3, p_4; t, \tau+t)\rangle$ on the variable τ in (4), since the timescale of the variation in $\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$ specified by this equation is of the order of $t \gg \tau$, while the derivatives of the function $\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$ with respect to the variable t are fairly smooth functions of t by virtue of the homogeneity of the medium.

Setting $\tau=0$ in (4), we can then successively construct equations [12] for functions of the type $\langle V^{\mu}(g)F(p_1 \pm g, p_2, p_3, p_4; t, t+\tau)\rangle$ which appear on the right-hand side of (4). Substituting the solution of the latter equations into (4), with $\tau=0$, and using the standard rules [12] for splitting up the correlation functions of the type

$$\langle V^{\mu}(g_1)V^{\nu}(g_2)F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle=\langle V^{\mu}(g_1)V^{\nu}(g_2)\rangle\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$$

which arise in the process, we find an equation for $\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$. Proceeding in the same way, we find an equation for the function $\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$ from (3) (but in this case with $\tau \neq 0$). Expanding the collision integral in the equations found for $\langle F(p_1, p_2, p_3, p_4; t)\rangle$ and $\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$ in the small momentum transfer g and also in $(\omega/E) \ll 1$—this is a legitimate step in the consideration of ultrarelativistic classically fast particles—we find the following equations for the functions $\langle F(p_1, p_2, p_3, p_4; t)\rangle$ and $\langle F(p_1, p_2, p_3, p_4; t, t+\tau)\rangle$ (a detailed derivation of (5) and (6) is given in the appendix):

$$\frac{\partial F_k(v_{\mu}, v_{\nu}, t, \tau)}{\partial \tau}-ikv_{\mu}(\eta)F_k(v_{\mu}, v_{\nu}, t, \tau)=\frac{q_{\mu}}{4}\frac{\partial^2}{\partial \eta^2}\{F_k(v_{\mu}, v_{\nu}, t, \tau)\} \tag{5}$$

$$\frac{\partial F_k(v_{\mu}, v_{\nu}, t, 0)}{\partial t}-ik(v_{\mu}(\eta)-v_{\nu}(\zeta))F_k(v_{\mu}, v_{\nu}, t, 0)$$

$$=\frac{q_0}{4}\left(\kappa_{\mu}\frac{\partial}{\partial \eta}+\kappa_{\nu}\frac{\partial}{\partial \zeta}\right)^2\left\{F_k(v_{\mu}, v_{\nu}, t, 0)\right\}. \tag{6}$$

Here $F_k(\boldsymbol{v}_\mu, \boldsymbol{v}_\nu, t, \tau)$ is the two-time distribution function in the k-representation, which is found from $\langle F(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4; t, t+\tau)\rangle$ by making the change of variables $\boldsymbol{p}_{1,2} = \boldsymbol{p}_\mu \mp \boldsymbol{k}/2, \boldsymbol{p}_{3,4} = \boldsymbol{p}_\nu \pm \boldsymbol{k}/2$. The quantities

$$\boldsymbol{v}_\mu, \boldsymbol{v}_\nu, \boldsymbol{p}_\mu, \boldsymbol{p}_\nu (q_0 = 2n_0 p_0^{-2} v_0^{-1} \sum_a a |U_0(\boldsymbol{a})|^2 \delta(E_{p_0} - E_{p_0 - a}))$$

are the velocities and momenta of particles μ and ν, and q_0 is the mean square value of the multiple-scattering angle per unit path length [13] of an elementary positive charge $e_0 > 0$; p_0 and $U_0(\boldsymbol{a})$ are its momentum at the time entrance into the medium and the Fourier component of the interaction potential of this charge with an isolated centre, and n_0 is the concentration of the scattering centres in the medium. The parameter $q_\mu \equiv q_0 \kappa_\mu^2$ is the mean square of the multiple-scattering angle per unit of path length of the particle having the charge e_μ and the energy E_μ, moreover $\kappa_\mu = U^\mu(g) p_0 p_\mu^{-1} U_0^{-1}(g)$. The angular vectors η and ζ satisfy the equations

$$\begin{aligned} \boldsymbol{v}_\mu &= v_0 \boldsymbol{e}_z (1 - \boldsymbol{\eta}^2/2) + v_0 \boldsymbol{\eta} \qquad & \boldsymbol{e}_z \boldsymbol{\eta} &= 0 \qquad & |\boldsymbol{\eta}| &\ll 1 \\ \boldsymbol{v}_\nu &= v_0 \boldsymbol{e}_z (1 - \zeta^2/2) + v_0 \boldsymbol{\zeta} \qquad & \boldsymbol{e}_z \boldsymbol{\zeta} &= 0 \qquad & |\boldsymbol{\zeta}| &\ll 1. \end{aligned} \tag{7}$$

Here $\boldsymbol{e}_z$ is the unit vector along the inward normal to the boundary of the medium.

Expanding the scalar products $\boldsymbol{k}\boldsymbol{v}_\mu$ and $\boldsymbol{k}\boldsymbol{v}_\nu$ on the left sides of (5) and (6) in the small quantities $|\boldsymbol{\eta}|$, $|\boldsymbol{\zeta}|$, $|\boldsymbol{\theta}_k|$($\boldsymbol{\theta}_k$ is an angular vector associated with the emission angle θ_k and the wave vector by equations like (7) (with $v_0 \to k$), we find the following expression for the function $F_k(\boldsymbol{\eta}, \boldsymbol{\zeta}, t, \tau)$ from (5) and (6):

$$\begin{aligned} F_k(\boldsymbol{\eta}, \boldsymbol{\zeta}, t, \tau) = \int \mathrm{d}^2\boldsymbol{\eta}'' \int \mathrm{d}^2\boldsymbol{\eta}' \int \mathrm{d}^2\boldsymbol{\zeta}' \cdot G_k^\mu(\boldsymbol{\eta} - \boldsymbol{\eta}'', \tau) \\ \times G_k^{\mu\nu}[(2\kappa_\mu)^{-1}(\boldsymbol{\eta}'' - \boldsymbol{\eta}') + (2\kappa_\nu)^{-1}(\boldsymbol{\zeta} - \boldsymbol{\zeta}'); \kappa_\mu^{-1}(\boldsymbol{\eta}'' - \boldsymbol{\eta}') \\ - \kappa_\nu^{-1}(\boldsymbol{\zeta} - \boldsymbol{\zeta}'); t; 0] \mathscr{P}_k(\boldsymbol{\eta}', \boldsymbol{\zeta}'). \end{aligned} \tag{8}$$

Here $\mathscr{P}_k(\boldsymbol{\eta}, \boldsymbol{\zeta})$ is the Fourier component of the particle distribution function at the time of entrance into the medium ($t = \tau = 0$), and $G_k^\mu(\tau)$, $G_k^{\mu\nu}(t, 0)$ is the Green function of (6) and (7) (see the appendix).

We then find the following result for the expectation value (over the positions of the scattering centres) of the bilinear combination of matrix elements of the transition current in the expression for the bremsstrahlung energy density in the medium:

$$\langle (\boldsymbol{n} \times \boldsymbol{j}_{i \to f}^\mu(\boldsymbol{k}, t_1)) [\boldsymbol{n} \times (\boldsymbol{j}_{i \to f}^\nu(\boldsymbol{k}, t_2))^*] \rangle = (\theta_k^2 + \boldsymbol{\eta}\boldsymbol{\zeta} - \boldsymbol{\theta}_k \boldsymbol{\eta} - \boldsymbol{\theta}_k \boldsymbol{\zeta}) F_k(\boldsymbol{\eta}, \boldsymbol{\zeta}, t, \tau) \tag{9}$$

where $F_k(\boldsymbol{\eta}, \boldsymbol{\zeta}, t, \tau)$ is determined by (7) and (8).

3. The spectral distribution of the bremsstrahlung by a system of classically fast charged particles in a scattering medium

Averaging $\mathrm{d}\varepsilon_\omega/\mathrm{d}\omega$ given by (1) over the positions of the scattering centres in the medium we find (taking into account (7)–(9)) the spectral distribution of the emission energy by a system of classically fast charged particles in matter:

$$\frac{dE_\omega}{d\omega} \equiv \left\langle \frac{d\varepsilon_\omega}{d\omega} \right\rangle = \frac{\omega^2 v_0^2}{2\pi^2} \sum_{\mu,\nu=1}^{N} e_\mu e_\nu \int_0^T dt \int_0^\infty d\tau \int d\Omega_n \int d^2\eta \int d^2\zeta \times (\theta_k^2 + \eta\zeta - \theta_k\eta - \theta_k\zeta) \int d^2\eta'' \int d^2\eta' \int d^2\zeta' G_k^\mu(\eta - \eta'', \tau) \times G_k^{\mu\nu}[(2\kappa_\mu)^{-1}(\eta'' - \eta') + (2\kappa_\nu)^{-1}(\zeta - \zeta'); \kappa_\mu^{-1}(\eta'' - \eta') - \kappa_\nu^{-1}(\zeta - \zeta'); t, 0]\mathscr{P}_k(\eta', \zeta') \exp(-i\omega\tau). \tag{10}$$

The above expression solves the problem of the calculation of the emission spectrum of the investigated system of particles because it determines $dE_\omega/d\omega$ by the Fourier component of the two-time distribution function $\mathscr{P}_k(\eta, \zeta)$ of irradiating particles at the time of entrance into a medium and by the parameters characterizing the interaction of the particles with atoms of the medium.

We assume that the system of classically fast charged particles consists of N identical particles which are flying into a scattering medium with $E_\mu = E_\nu = E$ at $t = \tau = 0$. In this case we are putting

$$\mathscr{P}_k(\eta, \zeta) = \delta(\eta - \Delta_\mu)\delta(\eta - \Delta_\nu) \exp(ikd_{\mu\nu}) \qquad \kappa_\mu^2 q_0 = \kappa_\nu^2 q_0 \equiv q \qquad d_{\mu\nu} = r_{0\mu} - r_{0\nu}.$$

On integrating over all $\eta, \eta', \eta''; \zeta, \zeta'$ using (9), (10), (28) and (29), we obtain the following expression for the spectral distribution of the emitted energy:

$$\frac{dE_\omega}{d\omega} = N\left(\frac{dE_\omega}{d\omega}\right)_1 + \frac{e^2\omega^2 v_0^2}{2\pi q} \mathrm{Re} \sum_{\mu \neq \nu = 1}^{N} \int_0^T dt \int_0^\infty \frac{ds \exp[-(1+i)s\xi^2\chi/2]}{at \cosh(s) A^2(s)} \times \left\{1 - \frac{\omega^2}{4A(s)}\left[((d_{\mu\nu})_\perp + v_0 t b_{\mu\nu})^2 + 2i(d_{\mu\nu})_z((d_{\mu\nu})_\perp + v_0 t b_{\mu\nu}) \frac{\partial}{\partial B} - (d_{\mu\nu})_z^2\left(\frac{\partial}{\partial A_0} - \frac{\partial^2}{\partial A_0^2}\right)\right] + \frac{i\omega}{2} b_{\mu\nu}\left((d_{\mu\nu})_\perp + v_0 t b_{\mu\nu} - (d_{\mu\nu})_z \frac{\partial}{\partial B}\right)\right\}\left[A_0^{-1} \exp\left(-\frac{B^2}{4A_0} + C\right)\right]. \tag{11}$$

Here

$$a = (i\omega q v_0/2)^{1/2} \qquad \chi = (\omega/qv_0)^{1/2} \qquad \xi = mE^{-1} \qquad b_{\mu\nu} = \Delta_\nu - \Delta_\mu$$

$$d_{\mu\nu} = (d_{\mu\nu})_\perp + e_z(d_{\mu\nu})_z \qquad A = \frac{i\omega}{2}(d_{\mu\nu})_z + aq^{-1} \tanh(s)$$

$$A_0 = (qt)^{-1} + \frac{ia\omega}{2} q^{-1} A^{-1}(s)(d_{\mu\nu})_z \tanh(s)$$

$$B = -\frac{i\omega^2 (d_{\mu\nu})_z}{2A(s)}((d_{\mu\nu})_\perp + v_0 t b_{\mu\nu}) + \omega\left((d_{\mu\nu})_\perp + \frac{v_0 t}{2} b_{\mu\nu}\right) - \frac{2i\Delta_\nu}{qt} \tag{12}$$

$$C = -\frac{\omega^2}{4A(s)}\left((\boldsymbol{d}_{\mu\nu})_\perp + v_0 t \boldsymbol{b}_{\mu\nu}\right)^2 - \frac{q\omega^2 v_0^2 t^3 \boldsymbol{b}_{\mu\nu}^2}{48} - \frac{\Delta_\nu^2}{qt} - \frac{\mathrm{i}\omega v_0 t}{2}\boldsymbol{b}_{\mu\nu}\Delta_\mu + \mathrm{i}(\boldsymbol{d}_{\mu\nu})_z\omega$$

$$\mathrm{e}_\mu = \mathrm{e}_\nu \equiv \mathrm{e}$$

$$\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1 = \frac{e^2\omega T\xi^2}{\pi}\left(\int_0^\infty \frac{\mathrm{d}s\,\exp(-\xi^2\chi s/2)\sin(\xi^2\zeta s/2)}{\tanh(s)} - \frac{\pi}{4}\right).$$

In the very long-wavelength range of the spectrum, $\omega \lesssim q\xi^{-4}$, we set $\tanh(s)=1$, retaining the main terms in the integrand in (11) as $\omega\xi^4 q^{-1} \ll 1$, and we then have

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega} = N^2 e^2 \frac{(q\omega)^{1/2} T}{\pi}\left\{1 + \mathrm{O}\left[\left(\frac{\omega\xi^4}{q}\right)\right]\right\}. \tag{13}$$

From this last equation it follows that in the range of very low frequencies ($\omega \to 0$) a system of ultrarelativistic charged particles radiates under total coherence conditions $\mathrm{d}E_\omega/\mathrm{d}\omega \sim N^2$.

In the opposite limiting case of the very high frequency

$$\omega \gg \max\{q\xi^{-4}; |\boldsymbol{d}_{\mu\nu}|_\perp^2 [(\boldsymbol{b}_{\mu\nu})^2 (\boldsymbol{d}_{\mu\nu})_z q T^3]^{-1}\}$$

we find, expanding the hyperbolic functions in small $S \ll 1$ in the integrand of (11) and retaining in the factor of the exponential and in its exponent the main terms for $\omega \to \infty$,

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega} = N\frac{2e^2 qT}{3\pi\xi^2} - \frac{4e^2}{q(2\pi)^{1/2}}\mathrm{Re}\left[\sum_{\mu\neq\nu=1}^{N}\int_{\tau_{\max}}^{T}\frac{\mathrm{i}\,\mathrm{d}t}{t(\boldsymbol{d}_{\mu\nu})_z^2}\right.$$
$$\times\left(\frac{((\boldsymbol{d}_{\mu\nu})_\perp^2 + v_0 t\boldsymbol{b}_{\mu\nu})^2}{(\boldsymbol{d}_{\mu\nu})_z} + \boldsymbol{b}_{\mu\nu}((\boldsymbol{d}_{\mu\nu})_\perp + v_0 t\boldsymbol{b}_{\mu\nu})\right) z_0^{-1/2}$$
$$\left.\times\exp\left(\mathrm{i}z_0 + \mathrm{i}\omega(\boldsymbol{d}_{\mu\nu})_z - \frac{q\omega^2 v_0^2 t^3 (\boldsymbol{b}_{\mu\nu})^2}{48}\right)\right\} \tag{14}$$

where $z_0 = \mathrm{i}\omega t|\boldsymbol{b}_{\mu\nu}|(v_0\xi^2/2)^{1/2}$, and $\tau_{\max} = q^{-1}\xi^2$ is the maximal time for the given frequency range. It follows from (14) that in the high-frequency range the interference terms decrease as ω increases and $\mathrm{d}E_\omega/\mathrm{d}\omega$ becomes equal to the energy of the radiation of N independent particles.

In the case when the characteristic longitudinal (in the direction of the particle motion) size of the beam is such that $\min\{(\boldsymbol{d}_{\mu\nu})_z\} \gg \max\{\omega^{-1}, \tau\}$ (but, of course, $\max\{(\boldsymbol{d}_{\mu\nu})_z\} \sim l_{\mathrm{B}} \ll v_0 T$) the terms in the sum on the right-hand side of (11) are periodic functions of the frequency. The inequality

$$\left|\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_{\mathrm{extr}} - N\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1\right|\left[\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1 N\right]^{-1} \sim \frac{\ln(T\tau_{\max}^{-1})}{qT\omega(\boldsymbol{d}_{\mu\nu})_z}(N-1) \lesssim \frac{\tau_{\max}}{T}(N-1)\ln(T\tau_{\max}^{-1}) \ll 1$$

is then satisfied for any ω. For a sufficiently extended beam of emitting particles the interference effects thus turn out to be of little importance and the spectral distribution of the emission energy $\mathrm{d}E_\omega/\mathrm{d}\omega$ as a function of the frequency ω basically repeats the behaviour of the function $(\mathrm{d}E_\omega/\mathrm{d}\omega)_1$ in its dependence, which occurs for an individual particle [2, 3, 5].

In the opposite limiting case of small $(\boldsymbol{d}_{\mu\nu})_z$ ($\max(\boldsymbol{d}_{\mu\nu})_z \ll \tau\xi^2$), we have, putting all $(\boldsymbol{d}_{\mu\nu})_z = 0$ in (11) and (12), after some simple transformations

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega} = -\frac{e^2\omega\xi^2}{\pi}\,\mathrm{Im}\Bigg(\sum_{\mu,\nu=1}^{N}\int_0^T \mathrm{d}t\,\exp\bigg\{\frac{\mathrm{i}\omega v_0 t}{2}\,\boldsymbol{b}_{\mu\nu}^2 - \frac{q\omega^2 v_0^2 t^3}{48}\,\boldsymbol{b}_{\mu\nu}^2$$

$$-qt\omega^2\frac{((\boldsymbol{d}_{\mu\nu})_\perp + v_0 t\boldsymbol{b}_{\mu\nu}/2)^2}{4} + \mathrm{i}\omega(\boldsymbol{d}_{\mu\nu})_\perp\boldsymbol{\Delta}_\nu$$

$$\times\bigg\{\bigg[\int_0^\infty \frac{\mathrm{d}S}{\tanh(s)}\exp\bigg(-\frac{(1+\mathrm{i})S\chi\xi^2}{2} - \gamma\coth(s)\bigg) + \frac{\mathrm{i}\pi}{4}\,\delta_{\mu\nu}\bigg] - a\xi^2$$

$$\times\int_0^\infty \frac{\mathrm{d}S}{\sinh(s)}\exp\bigg(-\frac{(1+\mathrm{i})S\chi\xi^2}{2} - \gamma\coth(s)\bigg)\boldsymbol{b}_{\mu\nu}((\boldsymbol{d}_{\mu\nu})_\perp + v_0 t\boldsymbol{b}_{\mu\nu})\bigg\}\bigg\}\Bigg)$$

$$\gamma = q\omega^2((\boldsymbol{d}_{\mu\nu})_\perp + v_0 t\boldsymbol{b}_{\mu\nu})/4a. \tag{15}$$

For different frequency ranges we have the following expression for the radiation energy:

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega} = N^2\frac{e^2(q\omega)^{1/2}T}{\pi} \qquad \omega \lesssim q\xi^{-4}. \tag{16}$$

In another extreme case $\omega \gg q\xi^{-4}$ we obtain

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega} = N\frac{2e^2qT}{3\pi\xi^2} + \frac{e^2q}{(2\pi\xi)^{1/2}}\sum_{\mu\neq\nu=1}^{N}\int_0^T \mathrm{d}t$$

$$\times\exp\bigg[-\frac{q\omega^2t^3}{48}\,\boldsymbol{b}_{\mu\nu}^2 - \frac{q\omega^2 t}{4}\bigg((\boldsymbol{d}_{\mu\nu})_\perp + \frac{v_0 t\boldsymbol{b}_{\mu\nu}}{2}\bigg)^2\bigg]$$

$$\times\,\omega^{3/2}\bigg|(\boldsymbol{d}_{\mu\nu})_\perp + \frac{v_0 t}{2}\,\boldsymbol{b}_{\mu\nu}\bigg|^{3/2}\exp(-\omega\xi|(\boldsymbol{d}_{\mu\nu})_\perp + v_0 t\boldsymbol{b}_{\mu\nu}|)_{\omega\to\infty}$$

$$\to N\frac{2e^2qT}{3\pi\xi^2} \equiv \bigg(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\bigg)_{\mathrm{BH}}. \tag{17}$$

Here $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{BH}} = 2e^2qT(3\pi\xi^2)^{-1}$ is the Bethe–Haitler emission energy [3].

Since the second term on the right-hand side of (17) is non-negative at any ω, it follows from asymptotic expressions in (16) and (17) that the frequency distribution of the bremsstrahlung of a system of non-interacting particles always has at least one extremum. This result is in contrast to the result for an individual radiator [1–3], in which case the bremsstrahlung energy spectrum in the medium is a monotonically increasing function of ω.

For further study of the radiation spectrum of a system of the particles in a scattering medium we specify the beam geometry. Below we consider in detail the spectral distribution of the radiation energy of a single-direction pulsed beam $(\boldsymbol{d}_{\mu\nu})_z = 0$, $|\boldsymbol{\Delta}_\mu| = 0$ of ultrarelativistic particles. This situation is especially singled out because for $(\boldsymbol{d}_{\mu\nu})_z = 0$, $|\boldsymbol{\Delta}_\mu| = 0$ there is in the initial beam neither a spatial distribution of particles in the propagation direction of the radiation nor a 'spread' in velocity for the particles. Interference effects which occur in this case are thus essentially dynamic, i.e. connected with the process of the passage of the particles through the scattering medium.

4. The spectral distribution of the radiation energy of a single-direction pulsed beam of ultrarelativistic particles

Since the distances between the particles in the initial beam are, as a rule, random quantities, to find the observed spectral density of the radiation energy it is necessary to average $\mathrm{d}E_\omega/\mathrm{d}\omega$ over all possible values of the vector $(\boldsymbol{d}_{\mu n})_\perp$. Putting in (11) and (12) all $(\boldsymbol{d}_{\mu\nu})_z$ and $|\Delta_\mu|$ equal to zero and averaging the obtained expressions over $(\boldsymbol{d}_{\mu\nu})_\perp$ along the pulsed beam cross-section (which we assume to be approximately a circle of diameter D) we obtain

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}=N\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1-N(N-1)\frac{4e^2\xi^2\omega}{\pi D^2}$$
$$\times\mathrm{Im}\left[\int_0^T\mathrm{d}t\int_0^\infty\frac{\mathrm{d}s}{\tanh(s)}\exp\left(-\frac{(1+\mathrm{i})\chi\xi^2 s}{2}\right)\right.$$
$$\left.\times\frac{4(1-\exp\{-D^2[qt\omega^2+q\omega^2/a\tanh(s)]/16\})}{qt\omega^2+q\omega^2/a\tanh(s)}\right].\tag{18}$$

where $(\mathrm{d}E_\omega/\mathrm{d}\omega)_1$ is the spectral energy density of the bremsstrahlung of an individual particle [3].

In the low-frequency range, $\omega\lesssim q\xi^{-4}$, putting $\tanh(s)=1$ in the exponent of (18), we have

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}=N\frac{e^2(q\omega)^{1/2}T}{\pi}+\frac{16e^2N(N-1)}{\pi\omega D^2(q\omega)^{1/2}}\left[\ln\frac{T}{\tau_q}-E_1\left(\frac{q\omega^2\tau_q D^2}{16}\right)+E_1\left(\frac{q\omega^2TD^2}{16}\right)\right]\tag{19}$$

where $E_n(S)$ is the exponential integral [14], $\tau_q=(q\omega)^{-1/2}$ is a characteristic time for the formation of radiation in the medium in the low-frequency range $\omega\lesssim q\xi^4$.

In the short-wavelength region of the spectrum, $\omega\gtrsim q\xi^{-4}$, we have, expanding the factor of the exponent and its index in (18) in terms of smalls and restricting ourselves to the main terms in $q\xi^{-4}\omega^{-1}\ll 1$,

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}=N\frac{2e^2qT}{3\pi\xi^2}+\frac{4e^2N(N-1)}{3\pi}\left\{\frac{8\ln(T\tau_{\max}^{-1})}{(D\omega\xi)^2}-K_2\left(\frac{\omega\xi D}{2}\right)\right.$$
$$\left.\times\left[E_1\left(\frac{q\omega^2D^2\tau_{\max}}{16}\right)-E_1\left(\frac{q\omega^2D^2T}{16}\right)\right]\right\}\tag{20}$$

where $K_n(s)$ is the modified Bessel function [15], $\tau_{\max}=q^{-1}\xi^2$ is the maximum of the characteristic times for the formation of radiation in the $\omega\gtrsim q\xi^{-4}$ frequency range considered.

The results in (19) and (20) show that $\mathrm{d}E_\omega/\mathrm{d}\omega$ is an increasing function of the frequency at $\omega\lesssim q\xi^{-4}$, and at $\omega\gtrsim q\xi^{-4}$ the emission energy $\mathrm{d}E_\omega/\mathrm{d}\omega$ decreases with increasing ω. It follows that the spectral energy density of the bremsstrahlung of a pulsed beam in a medium has a maximum, and this maximum is unique. Detailed analyses show if $qD\xi^{-3}\gg 1$, then $\omega_{\max}\sim q\xi^{-4}$, and the bremsstrahlung energy $\mathrm{d}E_\omega/\mathrm{d}\omega$ is of the same order of magnitude as the background due to the Bethe–Haitler radiation emission $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{BH}}=2e^2qT/3\pi\xi^2$. For $qD\xi^{-3}\lesssim\xi(qT)^{-1/2}\ll 1$ the maximum of the spectrum is again at the frequency $\omega_{\max}\sim q\xi^{-4}$, but in this case we have the ratio $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\max}:(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{BH}}\approx N$ (N is the number of radiating particles). If, on the other hand, the conditions $qD\xi^{-3}\ll\xi(qT)^{-1/2}\ll 1$ hold, the maximum is a plateau with a width equal in order of magnitude to $D^{-1}(qT)^{-1/2}$, and we have $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\max}(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{BH}}^{-1}\approx N$.

5. Radiation emission by a set of non-identical particles in scattering matter. Bremsstrahlung by an ultrarelativistic electron–positron pair

We note that the Green function $G_k^{\mu\nu}(x, y, t, 0)$ (see the appendix) which is in (10) is proportional to $\mathrm{Sinh}^{-1}\{[\mathrm{i}(\kappa_\mu^2-\kappa_\nu^2)]^{1/2}t\tau_q^{-1}\}$. Here $\tau_q=(q\omega)^{-1/2}$ is the characteristic time of coherence when the Landau–Pomeranchuck effect takes place. As to $|\kappa_\mu^2-\kappa_\nu^2|\gtrsim(\tau_q t^{-1})^2\sim(\tau_q T^{-1})^2\ll 1$, for any radiation frequency ω the contribution of the addends with $\mu\neq\nu$ to the spectral distribution (10) is very small compared with the emission energy of its own emission energy $(\mathrm{d}E_\omega/\mathrm{d}\omega)_0$. The calculations give us that the ratio of the interference part $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{interf}}$ of the emission energy to the value of its own $(\mathrm{d}E_\omega/\mathrm{d}\omega)_0$ emission energy is no more than

$$(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{interf}}(\mathrm{d}E_\omega/\mathrm{d}\omega)_0^{-1}\lesssim(N-1)\tau_{\max}(\omega)T^{-1}\ll 1.$$

Here $\tau_{\max}(\omega)$ is the maximum coherence time for the corresponding frequency range. In that way non-identity (with respect to the difference of electromagnetic characteristics) of irradiating particles leads to suppression of the interference mechanism forming the bremsstrahlung spectrum of the considered system of emitting particles.

However, it should be noted that the inequality

$$|\kappa_\mu^2-\kappa_\nu^2|\gtrsim(\tau_q/T)^2\ll 1$$

which is true when the system of non-identical particles takes place, is infringed for an electron–positron pair. In this case

$$q_\mu=q_\nu=q_0 \qquad \kappa_{\mathrm{el}}=-\kappa_{\mathrm{pos}}=-1.$$

If the distance between an electron and a positron at the moment when they are flying into a medium ($t=0$) is equal to $|\boldsymbol{d}_{\mu\nu}|\equiv|\boldsymbol{d}|$ and their velocities are $\boldsymbol{v}_{\mathrm{el}}=\boldsymbol{v}_{\mathrm{pos}}=v_0\boldsymbol{e}_z$, we have for the emission energy of the ultrarelativistic electron–positron pair in the scattering medium the following expression:

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}=2\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1-\frac{e_0^2\omega^2}{\pi q_0}\mathrm{Re}\int_0^T\frac{\mathrm{d}t}{t}\int_0^\infty\frac{\mathrm{d}\tau\exp[-(\mathrm{i}\tau\xi^2/2)-(\omega^2d_\perp^2/4A_3(\tau))+\mathrm{i}\omega d_z]}{A_2(\tau)A_3(\tau)\cosh^2(a_0\tau)}$$
$$\times\left(\frac{(q_0t)^{-2}+a_0q_0^{-2}t^{-1}\tanh(a_0\tau)-A_2(\tau)(q_0\omega^2v_0^2t^3/12+\mathrm{i}\omega d_z/2)}{A_2^2(\tau)A_3(\tau)}\right.$$
$$\left.-\frac{\sigma(\tau)\omega^2d_\perp^2}{4A_3^2(\tau)}\right) \qquad \boldsymbol{d}=\boldsymbol{d}_\perp+d_z\boldsymbol{e}_z. \tag{21}$$

Here $A_2(\tau)$, $A_3(\tau)$, $\sigma(\tau)$ are given by the expressions

$$A_2(\tau)=(q_0t)^{-1}+a_0q_0^{-1}\tanh(a_0\tau) \qquad a_0=(iq_0\omega v_0/2)^{1/2}$$

$$A_3(\tau)=\frac{q_0\omega^2v_0^2t^3}{12}+\frac{i\omega d_z}{2}+\frac{(\omega v_0t)^2+4a_0q_0^{-2}t^{-1}\tanh(a_0\tau)}{4A_2(\tau)}-2a_0^3tq_0^{-2}A_2^{-1}(\tau)\tanh(a_0\tau)$$

$$\sigma(\tau)=\frac{4(q_0t)^{-2}+(\omega v_0t)^2+8a_0q_0^{-2}t^{-1}\tanh(a_0\tau)-8a^3tq_0^{-2}\tanh(a_0\tau)}{4A_2^2(\tau)}.$$

At $\omega\lesssim q_0\xi^{-4}$ when the characteristic time τ is about $(q_0\omega)^{-1/2}$ we obtain

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}=\frac{4}{3}\frac{e^2}{\pi}T^3(q_0\omega)^{3/2}\sim\omega^{3/2}. \tag{22}$$

It follows from the last expression that the interference of the waves irradiated by the electron and the positron in the medium leads to the suppression of the intensity of radiation as compared with the situation of an individual emitting particle $((\mathrm{d}E_\omega/\mathrm{d}\omega)_1\sim\omega^{1/2})$.

At $\omega\gtrsim q_0\xi^{-4}$, expanding all functions which are under the integral over τ in (21) by $|a_0|\tau\ll 1$, we obtain

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\approx 2\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1\left(1-\frac{12}{q_0\omega}\frac{1}{T\tau_{\max}}\right)_{\omega\to\infty}\to 2\left(\frac{\mathrm{d}E_\omega}{\mathrm{d}\omega}\right)_1. \tag{23}$$

Thus at large $\omega\lesssim q_0\xi^{-4}$ the interference effects are negligible and the emission energy is proportional to the number of irradiating particles.

It should be noted that at $\omega\lesssim q_0\xi^{-4}$ the spectral distribution $(\mathrm{d}E_\omega/\mathrm{d}\omega)$ has downwards convexity as a function of ω. But at $\omega\gtrsim q_0\xi^{-4}$ the $\mathrm{d}E_\omega/\mathrm{d}\omega$ is the function which is always convex upwards (at least at $d_z=0$). Therefore, at least in the case of a sufficiently small initial longitudinal distance d_z, the emission energy of the ultrarelativistic electron–positron pair (as a function of the radiation frequency) has a point of overbending at $\omega\sim q_0\xi^{-4}$.

6. Conclusion

We have constructed a consistent kinetic theory for the radiation of a system of classically fast non-interacting charged particles which undergo multiple elastic collisions in a scattering medium. We have found the spectral distribution of the radiation energy from such particles. The obtained spectrum depends strongly both on the parameters of the scattering medium, and on the characteristics of the initial system of the particles.

We have studied in detail the emission by the beam of identical particles. It is shown in this case that the spectral distribution of the emission energy has at least one extremum, in contrast to the situation for an individual particle [1–3] when the emission spectrum is a monotonic function from the emission frequency. If a pulsed beam of identical particles is considered, the extremum is then a maximum. Moreover, if the width of the initial beam D is such that the conditions $qD\xi^{-3}\ll\xi(qT)^{-1/2}\ll 1$ hold, the maximum of the bremsstrahlung energy spectrum is a plateau with a width of the order of $D^{-1}(qT)^{-1/2}$. The ratio of $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\max}$ to the background level (the energy of the Bethe–Haitler radiation $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{BH}}=2e^2qT/3\pi\xi^2$) is approximately equal to N, the number of radiating particles. As the parameter $qD\xi^{-3}$ increases, $qD\xi^{-3}\lesssim\xi(qT)^{-1/2}$, the plateau converts into a 'strict' maximum. As before, we have $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\max}(\mathrm{d}E_\omega/\mathrm{d}\omega)^{-1}_{\mathrm{BH}}\approx N$. If we have $qD\xi^{-3}\gg 1$, then the quantities $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\max}$ and $(\mathrm{d}E_\omega/\mathrm{d}\omega)_{\mathrm{BH}}$ become the same in order of magnitude.

The radiation emission by a system of non-identical particles has been considered. It is shown that the differences in the electrodynamic characteristics or irradiating particles lead to the suppression of the interference mechanism forming the emission spectrum. We have studied in detail the emission by an ultrarelativistic electron–positron pair in a scattering medium. We have shown that the interference of a wave emitted by the electron and the positron leads to the suppression of the intensity of the emission energy in the long-wave frequency range, while in the short-wave frequency region the interference effects are negligible and the value of the emission energy is proportional to the number of irradiating particles. We have shown that under some conditions the emission spectrum of the electron–positron pair has an overbending point which is located in the frequency range of the order of $q_0\xi^{-4}$.

Appendix

We write equations for the function on the right-hand side of (4) (for the case $\tau=0$). We use the standard rules [12] for breaking up the correlation functions of the type $\langle V^\mu(g)V^\nu(g_1)F(p_1,p_2,p_3,p_4;t,t+\tau)\rangle$ which arise in the process:

$$\langle V^\mu(g)V(g_1)F(p_1,p_2,p_3,p_4;t,t+\tau)\rangle = n_0U^\mu(g)(U^\nu(-g))^*\delta_{g,-g_1}\langle F(p_1,p_2,p_3,p_4;t,t+\tau)\rangle$$

where n_0 is the number of scattering centres per unit volume.

Solving the equations which result (see [11], for example), under the initial condition

$$\langle V^\mu(g)F(p_1,p_2,p_3,p_4;t,t+\tau)\rangle=\langle V^\mu(g)\rangle\langle F(p_1,p_2,p_3,p_4;t,t+\tau)\rangle=0$$

(this condition means that there are no correlations at the time $t=\tau=0$), we find the functions $\langle V^\mu(g)F(p_1+g,p_2,p_3,p_4;t)\rangle$,

$$\langle V^\mu(g)F(p_1,p_2-g,p_3,p_4;t)\rangle \qquad \langle V^\mu(g)F(p_1,p_2,p_3+g,p_4;t)\rangle$$
$$\langle V^\mu(g)F(p_1,p_2,p_3,p_4-g;t)\rangle.$$

Substituting the latter functions into the right-hand side of (4) with $\tau=0$, we find the equation for $\langle F(p_1,p_2,p_3,p_4;t,0)\rangle$. Proceeding in a similar way for the correlation function which appears on the right-hand side of (3), we find the equations for $\langle F(p_1,p_2,p_3,p_4;t,\tau)\rangle$. As a result we have

$$\begin{aligned}\frac{\partial}{\partial t}\langle F(p_1,p_2,p_3,p_4;t,0)\rangle&+\mathrm{i}(E_{p_1}-E_{p_2}+E_{p_3}-E_{p_4})\langle F(p_1,p_2,p_3,p_4;t,0)\rangle\\
&=-n_0\sum_g|U_0(g)|^2\{\delta_-(E_{p_1+g}-E_{p_2}+E_{p_3}-E_{p_4})[\kappa_\mu^2(\langle F(p_1,p_2,p_3,p_4;t,0)\rangle\\
&\quad-\langle F(p_1+g,p_2+g,p_3,p_4;t,0)\rangle)+\kappa_\mu\kappa_\nu(\langle F(p_1+g,p_2,p_3-g,p_4;t,0)\rangle\\
&\quad-\langle F(p_1+g,p_2,p_3,p_4+g;t,0)\rangle)]-\delta_-(E_{p_1}-E_{p_2-g}+E_{p_3}-E_{p_4})\\
&\quad\times[\kappa_\mu^2(\langle F(p_1-g,p_2-g,p_3,p_4;t,0)\rangle-\langle F(p_1,p_2,p_3,p_4;t,0)\rangle)\end{aligned}$$

$$+\kappa_\mu\kappa_\nu(\langle F(p_1,p_2-g,p_3-g,p_4;t,0)\rangle-\langle F(p_1,p_2-g,p_3,p_4+g;t,0)\rangle)]$$
$$+\delta_-(E_{p_1}-E_{p_2}+E_{p_3+g}-E_{p_4})[\kappa_\nu^2(\langle F(p_1,p_2,p_3,p_4;t,0)\rangle$$
$$-\langle F(p_1,p_2,p_3+g,p_4+g;t,0)\rangle)+\kappa_\mu\kappa_\nu(\langle F(p_1-g,p_2,p_3+g,p_4;t,0)\rangle$$
$$-\langle F(p_1,p_2+g,p_3+g,p_4;t,0)\rangle)]-\delta_-(E_{p_1}-E_{p_2}+E_{p_3}-E_{p_4-g})$$
$$\times[\kappa_\nu^2(\langle F(p_1,p_2,p_3-g,p_4-g;t,0)\rangle-\langle F(p_1,p_2,p_3,p_4;t,0)\rangle)$$
$$+\kappa_\mu\kappa_\nu(\langle F(p_1-g,p_2,p_3,p_4-g;t,0)\rangle$$
$$-\langle F(p_1,p_2+g,p_3,p_4-g;t,0)\rangle)]\} \quad (24)$$

$$\frac{\partial}{\partial\tau}\langle F(p_1,p_2,p_3,p_4;t,\tau)\rangle+\mathrm{i}(E_{p_1}-E_{p_2})\langle F(p_1,p_2,p_3,p_4;t,\tau)\rangle$$
$$=-n_0\kappa_\mu^2\sum_g|U_0(g)|^2\{\delta_-(E_{p_1+g}-E_{p_2})[\langle F(p_1,p_2,p_3,p_4;t,\tau)\rangle$$
$$-\langle F(p_1+g,p_2+g,p_3,p_4;t,\tau)\rangle]-\delta_-(E_{p_1}-E_{p_2-g})$$
$$\times[\langle F(p_1-g,p_2-g,p_3,p_4;t,\tau)\rangle-\langle F(p_1,p_2,p_3,p_4;t,\tau)\rangle]\}. \quad (25)$$

Here $\delta_-(x)$ is a one-side δ-function [14], $p_{1,2}=p_\mu\mp k/2$, $p_{3,4}=p_\nu\pm k/2$, and

$$F_k(p_\mu,p_\nu,t,\tau)\equiv\langle F(p_\mu-k/2;p_\mu+k/2;p_\nu+k/2,p_\nu-k/2,t,\tau)\rangle$$

is a two-time distribution function in the k-representation.

In obtaining (24) and (25) we take account of the fact that the timescales of the interaction of a particle with an isolated scattering centre, t_0, are small in comparison with t and τ. Therefore, in the zeroth approximation in $t_0\tau^{-1}\ll 1$ and in $t_0t^{-1}\ll 1$ we have replaced the functions $\langle F(p_1,p_2,p_3,p_4;t+t';\tau+\tau')\rangle$ by the functions $\langle F(p_1,p_2,p_3,p_4;t,\tau)\rangle$. In deriving (26) we note that the terms proportional to the correlation functions of the type $\langle V^\mu(g)F(p_1,p_2,p_3,p_4;t,0)\rangle$ have been discarded, since they are small quantities of the order of $\tau t^{-1}\ll 1$ with respect to the other terms in the equation.

Equations (24) and (25) describe the kinetics of the emission of bremsstrahlung photons in a scattering medium both in the definitely classical case ($E\gg\omega$) and in the quantum mechanical case, with $E\gtrsim\omega$. In this case the function $F_k(p_\mu,p_\nu,t,\tau)$ also depends on the spin variables $\lambda_1,\lambda_2,\lambda_3,\lambda_4$.

Taking the classical limit $E\gg\omega$, and expanding the functions $F_k(p_1\pm g;p_2\pm g;p_3\pm g;p_4\pm g;t,\tau)$ on the right-hand sides of (24) and (25) in the small quantity $|g|\ll|p_\mu|$ ($|p_\mu|$ is the momentum of the particle as it enters the medium), we find the following result in the Fokker–Planck approximation [13]:

$$\frac{\partial F_k(v_\mu,u_\nu,t,\tau)}{\partial\tau}-\mathrm{i}kv_\mu(\eta)F_k(v_\mu,u_\nu,t,\tau)=\kappa_\mu^2\frac{q_0}{4}\frac{\partial^2}{\partial\eta^2}F_k(v_\mu,v_\nu,t,\tau) \quad (26)$$

$$\frac{\partial F_k(v_\mu,v_\nu,t,0)}{\partial t}-\mathrm{i}k(v_\mu(\eta)-v_\nu(\xi))F_k(v_\mu,v_\nu,t,0)$$
$$=\frac{q_0}{4}\left(\kappa_\mu\frac{\partial}{\partial\eta}+\kappa_\nu\frac{\partial}{\partial\zeta}\right)^2F_k(v_\mu,v_\nu,t,0). \quad (27)$$

Here $v_\mu = p_\mu E_\mu^{-1}$; $v_\nu = p_\nu E_\nu^{-1}$; $q_0 = 2n_0 p_0^{-2} v_0^{-1} \Sigma_g g |U_0(g)|^2 \delta(E_{p_0} - E_{p-g})$ is the mean square multiple scattering angle per unit path length [13], and the angular vectors $\boldsymbol{\eta}$ and ζ are given by (7).

It is easy to find that the Green functions of (26) and (27) are given by the following expressions:

$$G_k^\mu(\boldsymbol{\eta}-\boldsymbol{\eta}'', \tau) = \frac{a_\mu}{\pi q_\mu \sinh(a_\mu \tau)} \exp\left[-\frac{a_\mu}{q_\mu} (\boldsymbol{\eta}-\boldsymbol{\eta}'')^2 \coth(a_\mu \tau) + \frac{2a_\mu}{q_\mu} \right.$$

$$\left. \times \tanh\left(\frac{a_\mu \tau}{2}\right)(\boldsymbol{\eta}-\boldsymbol{\eta}'')(\boldsymbol{\theta}_k - \boldsymbol{\eta}'') - \frac{2a_\mu}{q_\mu} \tanh\left(\frac{a_\mu \tau}{2}\right)(\boldsymbol{\theta}_k - \boldsymbol{\eta}'')^2 + \mathrm{i}kv_0\tau \right] \tag{28}$$

$$G_k^{\mu\nu}(\boldsymbol{\theta}-\boldsymbol{\theta}', \varphi_1 - \varphi', t, 0)$$

$$= \frac{a_{\mu\nu} \delta(\varphi' - \varphi_1)}{\pi q_0 \kappa_\mu^2 \kappa_\nu^2 \sinh(a_{\mu\nu} t)}$$

$$\times \exp\left[\frac{\mathrm{i}kv_0 t}{2} (\kappa_\mu + \kappa_\nu) \boldsymbol{\theta}_k \varphi_1 + \frac{\mathrm{i}kv_0 t}{2} (\kappa_\nu^2 - \kappa_\mu^2) \varphi_1^2 - \frac{a_{\mu\nu}}{q_0} \coth(a_{\mu\nu} t)(\boldsymbol{\theta}' - \boldsymbol{\theta}_1)^2 \right.$$

$$\left. + \frac{2a_{\mu\nu}}{q_0} \tanh\left(\frac{a_{\mu\nu} t}{2}\right)(\boldsymbol{\theta}_1 - \boldsymbol{\theta}')\boldsymbol{\theta}_k' - \frac{2a_{\mu\nu}}{q_0} \tanh\left(\frac{a_{\mu\nu} t}{2}\right) \boldsymbol{\theta}_k' + \frac{\mathrm{i}kv_0 t}{2} \boldsymbol{\theta}_k'^2 \right] \tag{29}$$

where we have introduced

$$a_\mu = (\mathrm{i}kv_0 q_\mu / 2)^{1/2} \qquad a_{\mu\nu} = (\mathrm{i}k' v_0 q_0 / 2)^{1/2} \qquad k' = k(\kappa_\mu^2 - \kappa_\nu^2)$$

$$\boldsymbol{\theta}(\boldsymbol{\eta}, \zeta) = \tfrac{1}{2}(\kappa_\mu^{-1} \boldsymbol{\eta} + \kappa_\nu^{-1} \zeta) \qquad \boldsymbol{\theta}_1 = \boldsymbol{\theta}(\boldsymbol{\eta}'', \zeta) \qquad \boldsymbol{\theta}' = \boldsymbol{\theta}(\boldsymbol{\eta}', \zeta')$$

$$\varphi(\boldsymbol{\eta}, \zeta) = \kappa_\mu^{-1} \boldsymbol{\eta} - \kappa_\nu^{-1} \zeta \qquad \varphi_1 = \varphi(\boldsymbol{\eta}'', \zeta) \qquad \varphi' = \varphi(\boldsymbol{\eta}', \zeta')$$

$$\boldsymbol{\theta}_k' = \frac{k}{k'} (\boldsymbol{\theta}_k(\kappa_\mu - \kappa_\nu) - \tfrac{1}{2}(\kappa_\mu^2 + \kappa_\nu^2)\varphi_1) - \boldsymbol{\theta}'.$$

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